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NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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148. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Find all the multiply perfect numbers of n different prime factors and of multiplicity $n-1$.

No solution received.

149. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Prove that every prime of the form $4n+1$ may be expressed as the sum of two parts r and s such that r^2+rs+s^2+1 is divisible by the prime.

I. Solution by E. B. ESCOTT, Ann Arbor, Mich.

Let $p=r+s=$a prime of the form $4n+1$. Substitute $s=p-r$ in $r^2+rs+s^2+1\equiv0$ (mod. p) and we have $r^2+r(p-r)+(p-r)^2+1\equiv0$ (mod. p) or $r^2+1\equiv0$ (mod. p), which is always possible when p is a prime of the form $4n+1$, and in many other cases.

Example. If $p=5$, $r^2+1\equiv0$ (mod. 5) has the solution $r\equiv2$ (mod. 5). $\therefore r=2$, $s=3$.

II. Solution by E. T. BELL, University of Washington.

When $p\equiv1$ (mod. 4) : $\left(\dfrac{-1}{p}\right)=1$ (Legendre's symbol).

Hence, for r a root of $x^2\equiv-1$ (mod. p), $rp-(r^2+1)\equiv0$ (mod. p); whence, putting $s=p-r$, $rs-1\equiv0$ (mod. p).

Therefore, $p^2-(rs-1)\equiv0$ (mod. p); or, r^2+rs+s^2+1 is a multiple of p; which establishes the stated theorem.

III. Solution by W. F. KING, Ottawa, Canada.

Since $r+s$ is a prime of form $4n+1$, it may be resolved in one way into the sum of two squares. Suppose it to be so resolved, and to be equal to h^2+k^2.

By the usual process, express h/k as a continued fraction in the form

$$\frac{h}{k}=x+\frac{1}{x_1+}\,\frac{1}{x_2+}\,\frac{1}{x_3+}\cdots$$

Form the successive convergents, and let the convergent next before the last (h/k), be (a/b).

Then $ak-bh=\pm1$.

And $(ah+bk)^2+(ak-bh)^2=a^2h^2+b^2k^2+a^2k^2+b^2h^2=(a^2+b^2)(h^2+k^2)$.

Therefore, $(ah+bk)^2+(ak-bh)^2$, or $(ah+bk)^2+1$, is divisible by h^2+k^2, or $r+s$.